LESSON 4.4c

Factor Theorem

Today you will:

- Use a special case of the Polynomial Remainder Theorem to factor polynomials
- Practice using English to describe math processes and equations

Prior Vocabulary:

- Factor
  - A factor of a number is a 2^{nd} number that divides evenly into the 1^{st} number (has a remainder of zero)
  - A factor of a polynomial is a 2^{nd} polynomial that divides evenly into the 1^{st} polynomial (has remainder zero).

- Polynomial Remainder Theorem, p. 176 (from lesson 4.3c)
  - To evaluate $f(x)$ when $x = a$ and find $f(a)$ … use synthetic division with $k = a$ … the remainder is $f(a)$.

- Remember…
  - When we ***solve*** a function or polynomial we are setting it equal to zero and finding its roots.
  - In other words we are finding its zeros, i.e. where the function $f(x) = 0$… where it has a zero remainder.

  - What if we use the Polynomial Remainder Theorem and find that the remainder is zero? We have something that divides evenly into the polynomial **… we have found a factor of the polynomial!**

Core Vocabulary: Factor Theorem

A polynomial $f(x)$ has a factor $x - a$ if and only if $f(a) = 0$ … i.e. if and only if a is a zero of $f(x)$

…this is a special application of the Polynomial Remainder Theorem.

How do we use this?

To tell if $x - a$ is a factor of $f(x)$, use synthetic division with $k = a$.

- if the remainder is zero, $(x - a)$ is a factor.

Don't forget, if it is easier you can just evaluate $f(a)$ … plug *a* into the function

- if $f(a) = 0$, $(x - a)$ is a factor.

Determine whether (a) $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$ and (b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

SOLUTION

…$f(x)$ is pretty simple...

STUDY TIP

In part (b), notice that direct substitution would have resulted in more difficult computations than synthetic division.

a. Find $f(2)$ by direct substitution.

$$f(2) = 2^2 + 2(2) - 4$$
$$= 4 + 4 - 4$$
$$= 4$$

Because $f(2) \neq 0$, the binomial $x - 2$ is not a factor of $f(x) = x^2 + 2x - 4$.

b. Find $f(-5)$ by synthetic division.

−5	3	15	−1	0	25
		−15	0	5	−25
	3	0	−1	5	0

Because $f(-5) = 0$, the binomial $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

Show that $x + 3$ is a factor of $f(x) = x^4 + 3x^3 - x - 3$. Then factor $f(x)$ completely.

SOLUTION

Show that $f(-3) = 0$ by synthetic division.

$$\begin{array}{r|rrrrr} -3 & 1 & 3 & 0 & -1 & -3 \\ & & -3 & 0 & 0 & 3 \\ \hline & 1 & 0 & 0 & -1 & 0 \end{array}$$

Because $f(-3) = 0$, you can conclude that $x + 3$ is a factor of $f(x)$ by the Factor Theorem. Use the result to write $f(x)$ as a product of two factors and then factor completely.

$f(x) = x^4 + 3x^3 - x - 3$	Write original polynomial.
$= (x + 3)(x^3 - 1)$	Write as a product of two factors.
$= (x + 3)(x - 1)(x^2 + x + 1)$	Difference of Two Cubes Pattern

ANOTHER WAY

Notice that you can factor $f(x)$ by grouping.

$f(x) = x^3(x + 3) - 1(x + 3)$

$= (x^3 - 1)(x + 3)$

$= (x + 3)(x - 1) \bullet (x^2 + x + 1)$

Determine if $x - 4$ is a factor of $f(x) = 2x^2 + 5x - 12$. If so, factor $f(x)$ completely.

SOLUTION

Determine if $f(4) = 0$ by synthetic division.

4	2	5	-12
		8	52
	2	13	40

Because $f(4) = 40$, you can conclude that $x - 4$ is NOT a factor of $f(x)$ by the Factor Theorem.

Determine if $x - 6$ is a factor of $f(x) = x^3 - 5x^2 - 6x$. If so, factor $f(x)$ completely.

SOLUTION

Show that $f(6) = 0$ by synthetic division.

$$\begin{array}{r|rrrr} 6 & 1 & -5 & -6 & 0 \\ & & 6 & 6 & 0 \\ \hline & 1 & 1 & 0 & 0 \end{array}$$

Because $f(6) = 0$, you can conclude that $x - 6$ is a factor of $f(x)$ by the Factor Theorem. Use the result to write $f(x)$ as a product of two factors and then factor completely.

$f(x) = (x - 6)(x^2 + x)$	Write original polynomial.
$= (x - 6)\cdot x(x + 1)$	Distributive property.
$= x(x - 6)(x + 1)$	Reorder…commutative property.

Homework

Pg 184, #39-54